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Full Length Research Paper

Empirical analysis of a micro-evolutionary algorithm for numerical optimization

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This paper presents an empirical comparison of some evolutionary algorithms to solve numerical optimization problems. The aim of the paper is to test a micro-evolutionary algorithm called Elitist evolution, originally designed to work with small populations, on a set of diverse test problems (unimodal, multimodal, separable, non-separable, shifted, and rotated) with different dimensionalities. The comparison covers micro-evolutionary algorithms based on differential evolution and particle swarm optimization. The number of successful runs, the quality of results and the computational cost, measured by the number of evaluations required to reach the vicinity of the global optimum, are used as performance criteria. Furthermore, a comparison against a state-of-the-art algorithm is presented. The obtained results suggest that the Elitist evolution is very competitive as compared with other algorithms, especially in high-dimensional search spaces.

Key words: Optimization methods, nature-inspired algorithms, evolutionary computation, swarm intelligence.

INTRODUCTION

Evolutionary algorithms (EAs) for optimization have become an invaluable tool for a wide range of tasks such as optimization of financial portfolios (Yan et al., 2009), evolution of neural networks for walking robots (Valsalam and Miikkulainen, 2008), file compression (Kattan and Poli, 2008), scheduling of an aircraft engine maintenance (Kleeman and Lamont, 2005), improving parameters for an induction motor (Hassan et al., 2011), predicting the optimum surface roughness when cutting acrylic sheets with laser beam cutting (Noor et al., 2011), among others. The specific optimization problem tackled in this work is the global optimization problem, which can be defined as the task of finding the point x^* with the minimum objective function value $f(x^*)$ (assuming minimization). When solved by an EA, this task usually requires a considerable amount of time and computational resources. EAs sample solutions by emulating natural evolution and the survival of the fittest. In this way, EAs evolve a population of candidate solutions in order to improve them. It is a well-known fact that EAs usually require large populations (Eiben and Smith, 2003). The reason is two-fold: (1) A large population enables the EA to explore more areas of the search space, and (2) it reduces the probability of premature nominal convergence. Nominal convergence occurs when all population individuals become very similar.

Micro-evolutionary algorithms (µ-EAs) are EAs designed to work with very small populations (Krishnakumar, 1989; Goldberg, 1989). Usually, a restart mechanism is employed each time the µ-EA reaches nominal convergence (due to its small population). µ-EAs, as described in the specialized literature, can be roughly divided into two classes: (1) Those that are modified versions of a traditional EA, and (2) Those specifically designed to work with small populations. Algorithms representative of the first class are the µ-genetic algorithm (µ-GA) (Krishnakumar, 1989) the µ-particle swarm optimization (µ-PSO) (Fuentes-Cabrera and Coello-Coello, 2007) and the µ-differential evolution (µ-DDE). On the other hand, an algorithm originally designed to work with a small population is the Elitist evolution (EEv) (Viveros-Jiménez et al., 2009)

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which was tested on some unconstrained optimization problems (Viveros-Jiménez et al., 2009) showing a competitive performance with respect to DE/rand/1/bin (Storn and Price, 1997) and, to the best of the authors' knowledge, there are two memetic algorithms with such feature: MGG+UNDX (Satoh et al., 1996) and G3+PCX (Deb et al., 2002). EEv's main features are as follows:

1. Elitism is integrated in the crossover and replacement operators.
2. Adaptive behavior: Elitism influences the step size used by the mutation operator, the number of individuals generated in each crossover and also the restart mechanism. Three adaptive parameters (not directly defined by the user) are used: (1) Ambient pressure $\mathrm{C} \in [1, \mathrm{P}]$, where *P* is the population size; (2) Step size for mutation operator, where each $b_j \in [0.0, 1.0]$, $j = 1, \ldots,$ N, N is the number of variables in the optimization problem, and (3) Crossover balance $CR \in [1, P-1]$.
3. The mutation operator works like a hill-climber search algorithm.
4. Two crossover operators allow an offspring to be a parent in the same generation. Parent selection is controlled by the adaptive parameter *C*, which changes the crossover operator behavior by allowing each member of the population to be chosen as a parent when a better solution was found. Otherwise, it forces the selection of the elite (the best solution so far) to be a parent.
5. The non-generational replacement mechanism is combined with a re-initialization mechanism. Either part or the whole population restarts at each generation; this mechanism is controlled by parameter *C*.
6. EEv employs two user-defined parameters: Population size (*P*≥*3*) and initial step size (*B*) required by the mutation operator.

EEv has the ability to search either locally (near a current point) or globally (on a distant point) according to the success of the optimization process. This ability is implemented through adaptive parameter *C* and the set of variation operators. The value of *C* is the number of individuals to be affected by a local search process. In this way, lower values of *C* promote global exploration while higher values of *C* promote local exploitation.

Despite the fact that there is a previous performance comparison of EEv against other EAs in a limited set of test functions (Viveros-Jiménez et al., 2009), no comparison against other µ-EAs has been reported in the specialized literature by using a more extended set of test problems.

The goal of this paper is then to test EEv on different types of optimization problems, where the sources of difficulty come from different dimensionalities and shifted and rotated functions. The performance of EEv is then compared with other µ-EAs and also with a state-of-the-art nature-inspired algorithm. Fourteen test problems with different dimensionality and linear transformations were used in the experiments. From the aforementioned motivation, the research question this paper aims to answer is that if EEv can outperform other µ-EAs and if it can be provide at least comparable results with respect to one state-of-the-art EA for global optimization.

MATERIALS AND METHODS

A µ-EA is defined as an EA that uses a small population size with a restart mechanism. The restart mechanism is employed to avoid premature convergence and to encourage exploration of the search space. µ-EAs have been used as optimizers for unconstrained (Krishnakumar, 1989), constrained (Fuentes-Cabrera and Coello-Coello, 2007) and multi-objective optimization problems (Toscano-Pulido and Coello-Coello, 2001). Additionally, a µ-EA can be used either as local improvement process (LIPs) to create efficient memetic algorithms (Kazarlis et al., 2001) or as part of cooperative evolutionary algorithms (Parsopoulos, 2009).

In this work, EEv (Viveros-Jiménez et al., 2009) and two other µ-EAs are tested on a set of global optimization problems. These two µ-EAs are the µ-Differential Evolution (Parsopoulos, 2009) and the µ-Particle Swarm Optimization (Fuentes-Cabrera and Coello-Coello, 2007). The three algorithms were implemented to report a direct comparison. Algorithms 1 to 3 present the details of the µ-Particle Swarm Optimization, the µ-Differential Evolution and EEv, respectively.

Two experiments are presented in this work. The first one aims to provide some insights into the behavior of EEv, other µ-EAs and one state-of-the-art EAs by means of a bird's eye analysis. The second one is an in-depth comparison on fourteen test problems with different features where four performance measures are employed as comparison criteria. The details of the two experiments, the corresponding results and a discussion of the findings are presented subsequently.

RESULTS AND DISCUSSION

Visual graphical comparison

A representative run on the Ackley's multimodal non-separable test function (f_{ack}) in 2D, is plotted at different stages of the search for a state-of-the-art EA and three µ-EAs: EEV in Figure 1, µ-PSO in Figure 2, µ-DE in Figure 3 and The Simple Adaptive Differential Evolution (SADE) (Qin et al., 2009) in Figure 4. The values of the parameters used for each algorithm are described in detail subsequently in "Performance Comparison". The following can be observed:

1. EEv required 210 FEs to locate its solutions in the neighborhood of the global optimum and 370 FEs to locate its individuals in the basin of the global optimum (Figure 1).
2. The µ-PSO was able to locate the neighborhood of the global optimum after 318 FEs and after 534 FEs, the population was in the basin of the global optimum (Figure 2).
3. The µ-DE required only 102 FEs to reach the vicinity of the global optimum and 210 Fes to locate its small

Algorithm 1. µ-PSO pseudocode.

Data: $P \in [3,6]$ (population size), $R > 0$ (Replacement generation), *N>0* (Number of restart particles), $M \in [0.0,1.0]$ (Mutation Rate), $C_1 \in \mathbf{R}$, $C_2 \in \mathbf{R}$, *Neighborhoods>0*.

Result: *Xbest* (best solution found).

```
Initialize particles' position, velocity and neighborhood randomly;
Set cont=1 and G=MaxFes/P;
For g=1 to G do
    If(cont==R)
        Reinitialization of N worst particles;
        Set cont=1;
    Recalculate best particles position $Xpbest_i$
    Select the local best position in the neighborhood $Lbest_i^g$;
    For each $X_i^g$, i=1, …,P do
        Recalculate particle speed
        Recalculate particle position;
    Perform mutation to each particle with a probability of P(M);
    Set cont=cont+1;
```

Algorithm 2. µ-DE pseudocode.

Data: $P \in [3,6]$ (population size), $CR \in \mathbf{R}$(Crossover Rate), $F \in \boldsymbol{R}$, $N \in \boldsymbol{N}$ (Number of restart solutions), $R \in \boldsymbol{N}$ (Replacement generation).

Result: *Xbest* (best solution found).

```
Set G=MaxFes/P, Cont=1;
For g=1 to G do
    If(cont==R)
        Reinitialization of N worst individuals;
        Set cont=1;
    Set $r_1 \neq r_2 \neq r_3,\ r_1, r_2, r_3 \in [1, P]\ , jrand \in [1, D]$ randomly;
    If($rndreal(0.0,1.0)_j < CR\ or\ j == jrand, j = 1, \dots, D, i = 1, \dots,\ P$)
        $U_{i,j} = X_{r1,j}^g + F \times \left(X_{r2,j}^g - X_{r3,j}^g\right)$;
    Else
        $U_{i,j} = X_{i,j}^g$;
    If($f\left(X_i^g\right) > f(U_i^g)$)
        $X_i^{g+1} = U_i^g$;
    Else
        $X_i^{g+1} = X_i^g$;
    Set cont=cont+1;
```

population in the basin of the global optimum (Figure 3).
4. SADE was able to locate its population in the neighborhood of the global optimum after 700 FEs, while after 1000 FEs all solutions were in the basin of the global optimum (Figure 4).

From this very general analysis, it can be observed that, of these three µ-EAs, µ-DE required fewer FEs to converge. The two algorithms which use adaptation (SADE and EEv) require more time to find the vicinity of the global optimum. Finally, in contrast to traditional population-based PSO, µ-PSO required more FEs to converge with respect to µ-DE and EEv.

These findings are indeed very general and not conclusive. They only intended to give insight into the behavior that we will analyze subsequently.

Performance comparison

In order to provide more solid evidences of the behavior

Algorithm 3. EEv pseudocode.

```
Data: $P \in [3,6]$ (population size), $B \in [0.0,1.0]$, R(initial stepsize).

Result: Xbest (best solution found).
Set $X^0$as a random initial population of size of P;
Set $b_j = B, j = 1, \dots, D$ as the initial stepsizes;
Set C=1,G=MaxFes/P;
For g=1 to G do
    Copy each $X^g$ individual in $O^g$
/*Mutation operator*/
    For each $O_i^g i = 1, \dots, P$ do
        Set $P_a = \frac{rnd\,(1,D)}{D}$;
        If($flip_{j\,(P_a)}, j = 1, \dots, D$)
            $O_{i,j}^g = X_{i,j}^g + rndreal(-b_j, b_j) \times (up_j - low_j)$;// Ensure that$O_{i,j}^g$"∈ $[low_j, up_j]$
/*Crossover operator*/
    For each $O_i^g i = 1, \dots, P$ do
        $k = rnd(0, P - C) + 1, l = rnd(0, P - C) + 1, m = rnd(1, P)$;
        $c_1 = rndreal(0.0,1.0), c_2 = rndreal(0.0,1.0 - c_1), c_3 = 1.0 - c_2 - c_1$;
        $O_i^g = c_1 \times O_k^g + c_2 \times X_l^g + c_3 \times O_m^g$;
/*Replacement*/
    Keep the C best individuals of $X^g \cup O^g$ and some random individuals from $O^g$;
    Set $Xbest^{g+1} = x_1^{g+1} + 1$;
/*Recalculation of adaptive parameters*/
    If($f(Xbest^g) > f(Xbest^{g+1})$)
        If(C>1)
            C=C-1;
        $b = |Xbest^{g+1} - Xbest^g/(up - low)|$;
    Else
        If(C<P)
            C=C+1;
        $b_j = rndreal(0.0, b_j), j = 0, \dots, D$;
    If (any $b_j$ value is equal to 0)
        Replace it with $B \times (1.0 - rndreal(0.0,1.0) \times \frac{gn}{G})$;
```

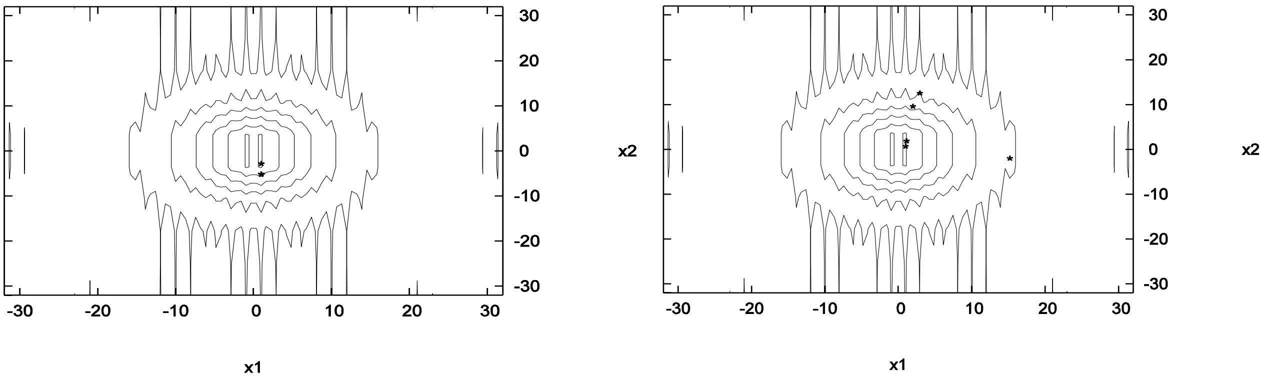
D is the problem dimensionality. *MaxFes* is the maximum allowed function evaluations. *Rnd (L,U)* returns a random integer value within *L* and *U*. *rndreal (L,U)* returns a random real value within *L* and *U*. up_j and low_j are the upper and lower bounds for the *j* dimension. *flip(P)* is a coin toss with a probability of *P*.

of the compared algorithms, the three different μ-EAs and SADE are further tested on a set of test problems with different features. DE/rand/1/bin and local-best PSO algorithms are also included, but just as a reference to μ-DE and μ-PSO, respectively.

We adopted the multicriteria statistical method proposed in (Carrano et al., 2011) to get a solid statistical foundation on our findings. The methodology defines a process where statistical tests are applied iteratively to independent samples of runs carried out by evolutionary algorithms used in an empirical comparison with the aim to rank them based on two or more criteria. The reader is referred to Carrano et al. (2011) for further details on the statistical methodology. Four criteria have been considered in our comparisons.

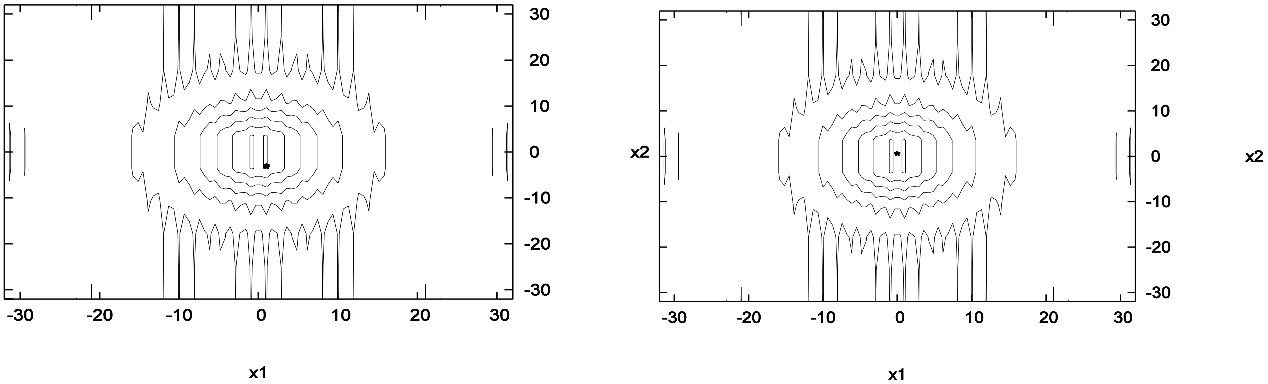
1. Best objective function value obtained in each algorithm run (denoted by best). Best = 0.0 means that the specified algorithm reaches the global optimum value with 1E-08 precision.
2. Number of function evaluations (FEs) required for reaching 95% of algorithm's improvement (denoted by IP).
3. Number of FEs spent by the algorithm before finding the global optimum value (denoted by speed). If the global optimum value was not found we set a value of 3e+5.
4. Success rate (denoted by success) calculated as the percentage of runs where the global optimum was reached out of the total number of independent runs carried out.

The fourteen benchmark functions are summarized in Table 1 and also detailed in the Appendix. 100 independent runs per algorithm per each test function were computed. The termination condition on each run was 300,000 FEs. We ranked the algorithms using one-way ANOVA over 100 30-quantiles extracted from the means of 1000 subsamples generated by bootstrapping. The ranking was validated using the permutation test as proposed by Carrano et al. (2011). We adopted a



(a) FEs = 170

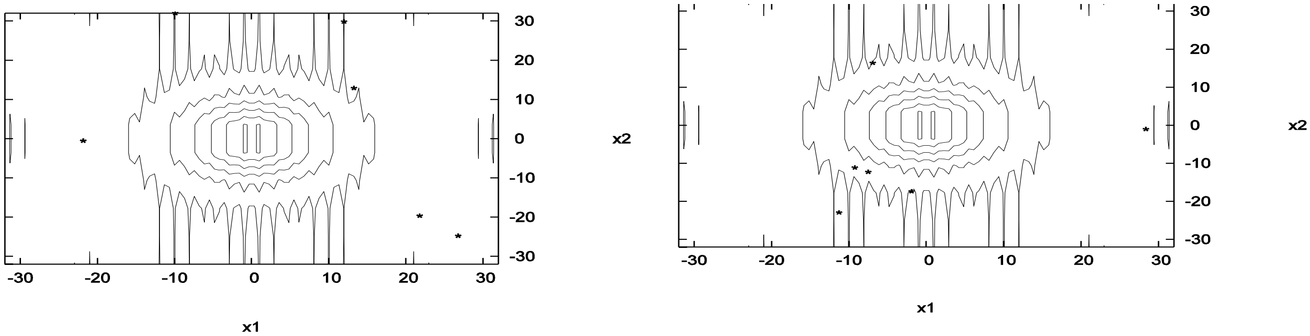
(c) Fes = 210



(b) FEs = 190

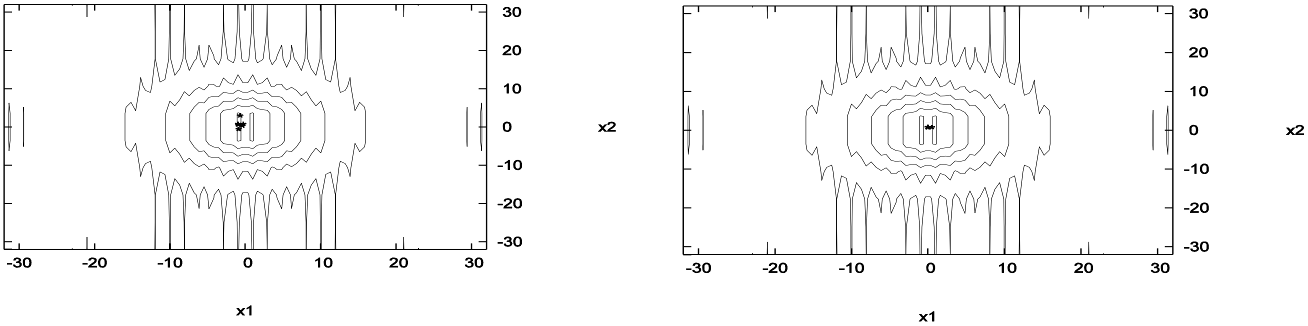
(d) Fes = 370

Figure 1. EEv sample run on 2-D f_{ack}. The figure shows the populations at different numbers of FEs.



(a) FEs = 102

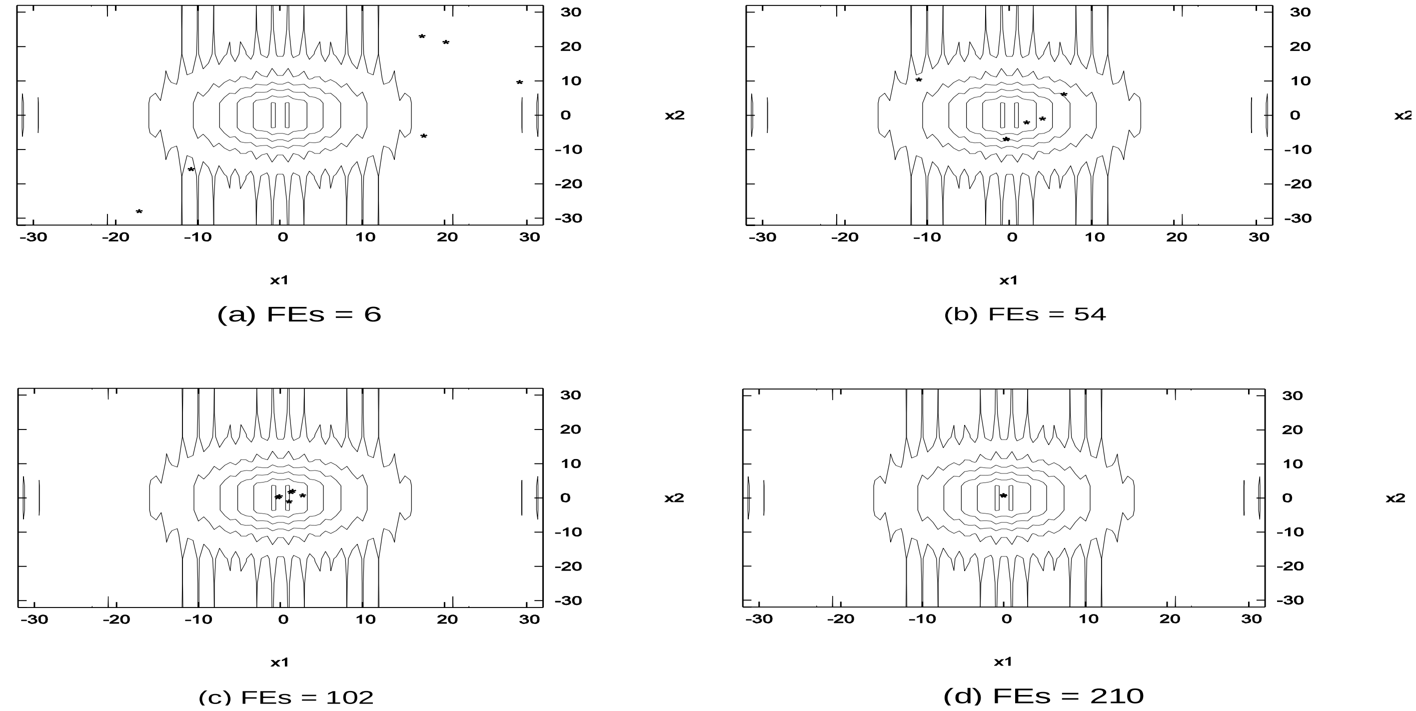
(b) FEs = 222



(c) FEs = 318

(d) FEs = 534

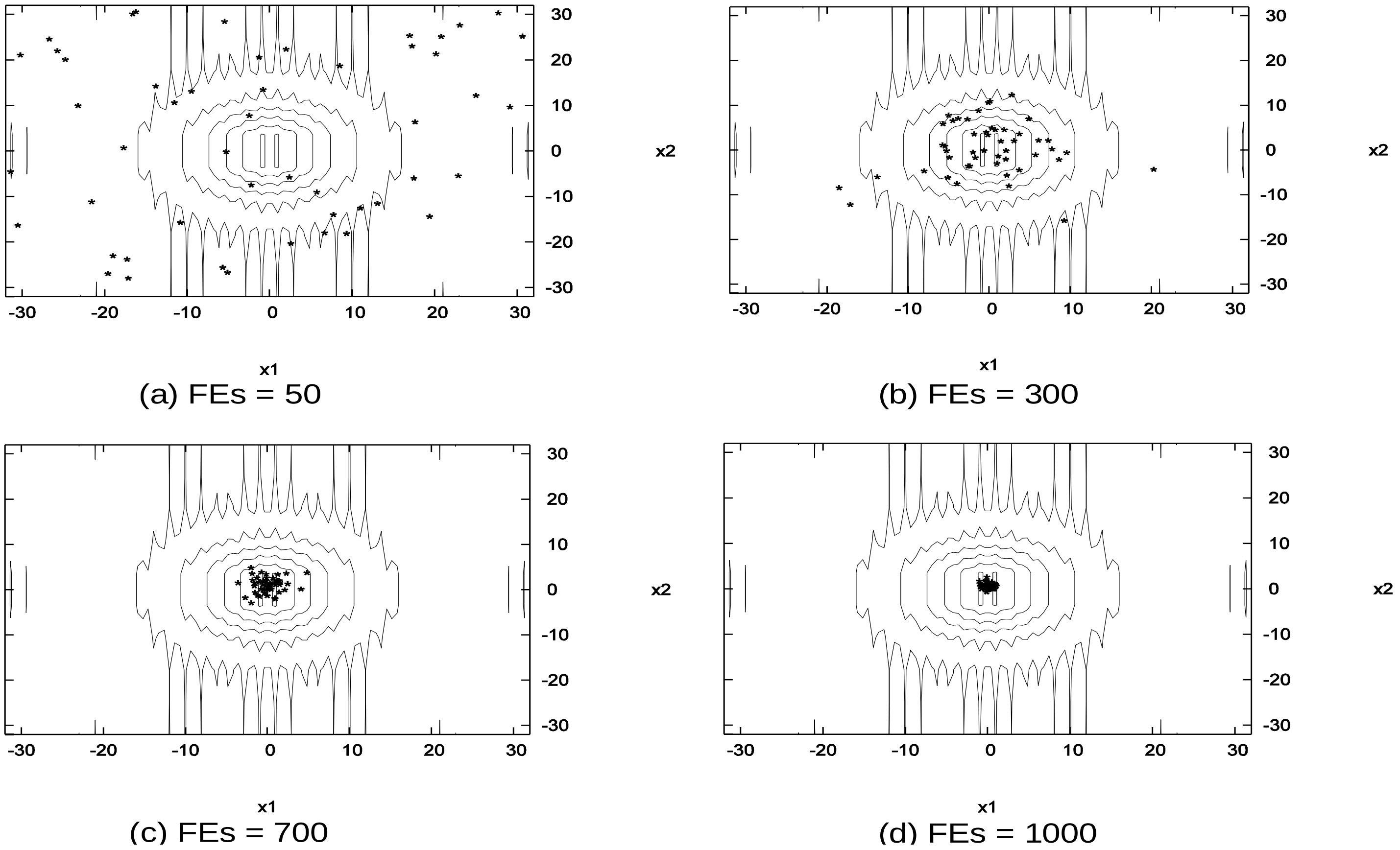
Figure 2. µ-PSO sample run on 2-D f_{ack}. Figure shows the populations at different numbers of FEs.



(a) FEs = 6 (b) FEs = 54

(c) FEs = 102 (d) FEs = 210

Figure 3. μ-DE sample run on 2-D f_{ack}. Figure shows the populations at different numbers of FEs.



(a) FEs = 50 (b) FEs = 300

(c) FEs = 700 (d) FEs = 1000

Figure 4. SADE sample run on 2-D f_{ack}. Figure shows the populations at different numbers of FEs.

Table 1. Test functions.

Variable	**Unimodal functions**
Separable	
f_{sph}	Sphere model
$f_{2.22}$	Schwefel's problem 2.22
$f_{2.21}$	Schwefel's problem 2.21
f_{stp}	Step function
f_{qtc}	Quartic function with noise
Non-separable	
$f_{1.2}$	Schwefel's problem 1.2
	Multimodal functions
Separable	
f_{sch}	Generalized Schwefel's problem 2.26
f_{ras}	Generalized Rastrigin's function
Non-separable	
f_{ros}	Generalized Rosenbrock's function
f_{ack}	Ackley's function
f_{grw}	Generalized Griewank's function
f_{sal}	Salomon's function
$f_{pen\,1,2}$	Generalized penalized functions

confidence level of 0.05 for the two statistical tests.

All the experiments were performed using a Pentium 4 PC with 512 MB of RAM. The algorithms were implemented in C language under a Linux environment. Three different dimensionalities were considered in the experiments, D = 30,100,200 for the non-rotated non-shifted test functions and for those shifted and rotated test functions D = 30 was used.

The following parameter values for each algorithm were chosen based on the suggestions found in their corresponding references, according to how their best performances were observed:

1. EEv: P = 5, B = 0.6.
2. μ-PSO: P = 6, $C_1 = C_2 = 1.8$, Neighborhoods = 2, Replacement generation = 100, replacement particles = 2. mutation % = 0.01, based on (Fuentes-Cabrera and Coello-Coello, 2007).
3. μ-DE: P = 6, CR = 0.1, F = 0.9, replacement generation = 100, replacement solutions = 2 (Parsopoulos, 2009).
4. Simple adaptive differential evolution: Set as suggested in (Qin et al. 2009).
5. PSO: P = 60, $C_1 = C_2 = 1.8$, neighborhoods = 10, mutation % = 0.01 (Fuentes-Cabrera and Coello-Coello, 2007).
6. DE: P = 30, CR = 0.9, F = 0.9, as in (Noman et al. 2008).

The obtained results are presented as follows: Tables 2 to 4 shows the results obtained on unimodal, multimodal non-separable and multimodal separable test function with 30D, respectively. Tables 5 to 7 show the results obtained on 30D shifted problems with multimodal separable, unimodal, and multimodal non-separable problems test functions, respectively. Tables 8 to 10 present the results obtained on 30D rotated problems with unimodal, multimodal non-separable, and multimodal separable test functions, respectively. Tables 11 to 13 include the results obtained on 100D problems with multimodal separable, unimodal and multimodal non-separable test functions, respectively. Finally, Tables 14 to 16 show the results obtained on 200D problems with unimodal, multimodal non-separable, and multimodal separable test functions, respectively.

All results are displayed in the following format:

	1.0	1.0	1.0	0.92	0.90	0.22
Criterion	**PSO** ≡	**μ-PSO** ≡	**EEv** >	**SADE** >	**DE** >	**μ-DE**

where *criterion* is the measured criterion, the *number* is the mean value computed for such *criterion*, the names of the algorithms are abbreviated and ordered from best to worst (left to right), and the logical operators indicate the result of the statistical tests. Therefore, in this example PSO, μ-PSO and EEv were tied as the best ranked approaches and μ-DE was the worst.

The discussion of results is presented as follows: "Performance analysis of each μ-EA" Subsection discusses the overall performance of each μ-EA (EEv, μ-DE and μ-PSO) on each different set of tests functions

Table 2. Multicriteria ranking of unimodal problems with D=30.

f_{sph}	**Best**	0.00E+0 DE ≡	0.00E+0 μ-DE ≡	0.00E+0 SADE ≡	0.00E+0 EEv ≡	0.00E+0 PSO ≡	0.00E+0 μ-PSO
	IP	3.90E+2 EEv <	1.19E+3 μ-DE <	1.72E+3 SADE <	1.82E+3 μ-PSO <	6.59E+3 PSO <	1.62E+4 DE
	Speed	1.89E+4 SADE <	3.65E+4 μ-DE <	5.04E+4 EEv <	8.43E+4 μ-PSO <	1.87E+5 DE <	2.24E+5 PSO
	Success	1.0 DE ≡	1.0 μ-DE ≡	1.0 SADE ≡	1.0 EEv ≡	1.0 PSO ≡	1.0 μ-PSO
$f_{2.22}$	**Best**	0.00E+0 DE ≡	0.00E+0 μ-DE ≡	0.00E+0 SADE ≡	0.00E+0 EEv ≡	0.00E+0 PSO ≡	0.00E+0 μ-PSO
	IP	1.57E+3 μ-DE <	2.07E+3 SADE <	4.05E+3 μ-PSO <	1.01E+4 PSO <	1.96E+4 DE <	2.75E+4 EEv
	Speed	2.53E+3 μ-DE <	3.08E+3 SADE <	9.44E+3 μ-PSO <	2.00E+4 PSO <	2.64E+4 DE <	4.79E+4 EEv
	Success	1.0 DE ≡	1.0 μ-DE ≡	1.0 SADE ≡	1.0 EEv ≡	1.0 PSO ≡	1.0 μ-PSO
$f_{2.21}$	**Best**	5.05E+0 SADE <	7.14E+0 EEv <	1.35E+1 DE <	1.73E+1 PSO <	1.77E+1 μ-PSO <	3.18E+1 μ-DE
	IP	1.15E+4 SADE <	4.67E+4 DE <	1.23E+5 μ-DE <	1.33E+5 μ-PSO <	1.67E+5 PSO <	1.76E+5 EEv
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{stp}	**Best**	0.00E+0 PSO ≡	0.00E+0 μ-PSO ≡	0.00E+0 μ-DE ≡	0.00E+0 EEv <	1.09E-1 <SADE	2.94E-1 DE
	IP	1.22E+3 μ-DE ≡	1.26E+3 EEv <	1.73E+3 SADE <	1.87E+3 μ-PSO <	6.75E+3 PSO <	1.60E+4 DE
	Speed	1.11E+4 μ-DE <	3.18E+4 μ-PSO ≡	3.37E+4 SADE <	8.10E+4 PSO <	1.10E+5 EEv ≡	1.12E+5 DE
	Success	1.0 μ-PSO ≡	1.0 μ-DE ≡	1.0 EEv ≡	1.0 PSO >	0.87 SADE >	0.82 DE
f_{qtc}	**Best**	0.00E+0 DE ≡	0.00E+0 μ-DE ≡	0.00E+0 SADE ≡	0.00E+0 EEv ≡	0.00E+0 PSO ≡	0.00E+0 μ-PSO
	IP	6.57E+1 EEv <	4.77E+2 μ-DE <	8.00E+2 μ-PSO <	8.82E+2 SADE <	3.22E+3 PSO <	7.69E+3 DE
	Speed	1.03E+4 EEv <	1.10E+4 SADE <	1.20E+4 μ-DE <	2.07E+4 μ-PSO <	6.83E+4 PSO <	9.13E+4 DE
	Success	1.0 DE ≡	1.0 μ-DE ≡	1.0 SADE ≡	1.0 EEv ≡	1.0 PSO ≡	1.0 μ-PSO
$f_{1.2}$	**Best**	4.72E-4 EEv <	1.51E-2 μ-PSO <	2.40E-2 SADE <	4.22E-2 DE <	8.82E+0 PSO <	1.21E+4 μ-DE
	IP	7.35E+2 EEv <	5.53E+3 μ-PSO <	7.58E+3 SADE <	1.14E+4 μ-DE <	3.28E+4 PSO <	5.04E+4 DE
	Speed	2.84E+5 SADE <	3.00E+5 μ-DE ≡	3.00E+5 DE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.32 SADE >	0.0 μ-DE ≡	0.0 DE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

Table 3. Multicriteria ranking of multimodal non-separable problems with D=30.

f_{ros}	**Best**	8.46E+0 PSO ≺	1.26E+1 DE ≺	1.75E+1 SADE≺	2.50E+1 μ-PSO≺	3.37E+1 EEv ≺	4.60E+1 μ-DE
	IP	8.72E+1 EEv ≺	5.27E+2 μ-DE≺	8.80E+2 μ-PSO≺	9.09E+2 SADE≺	2.99E+3 PSO≺	8.83E+3 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE≡	3.00E+5 SADE≡	3.00E+5 EEv≡	3.00E+5 PSO≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE≡	0.0 SADE≡	0.0 EEv≡	0.0 PSO≡	0.0 μ-PSO
f_{ack}	**Best**	0.00E+0 DE ≡	0.00E+0 μ-DE≡	0.00E+0 SADE≡	0.00E+0 EEv≡	0.00E+0 μ-PSO≺	4.46E-7 PSO
	IP	2.46E+3 EEv ≺	3.84E+3 SADE≺	3.88E+3 μ-DE≺	5.89E+3 μ-PSO≺	2.06E+4 PSO ≺	3.91E+4 DE
	Speed	2.72E+4 SADE≺	6.73E+4 μ-DE ≺	9.73E+4 EEv≺	1.78E+5 μ-PSO≺	2.65E+5 DE ≺	3.00E+5 PSO
	Success	1.0 DE ≡	1.0 μ-DE≡	1.0 SADE≡	1.0 EEv≡	1.0 μ-PSO≻	0.0 PSO
f_{grw}	**Best**	2.01E-3 PSO ≺	2.61E-3 DE ≺	3.53E-3 SADE≺	1.22E-2 μ-DE≺	1.78E-2 EEv ≺	2.00E-2 μ-PSO
	IP	4.42E+2 EEv ≺	1.57E+3 μ-DE≺	1.76E+3 SADE≺	1.79E+3 μ-PSO≺	6.80E+3 PSO≺	1.65E+4 DE
	Speed	6.89E+4 SADE≺	2.18E+5 DE ≺	2.28E+5 μ-PSO≺	2.35E+5 EEv≺	2.69E+5 PSO ≺	2.76E+5 μ-DE
	Success	0.87 PSO ≺	0.80 SADE≺	0.77 DE ≺	0.37 μ-PSO≺	0.28 EEv ≺	0.12 μ-DE
f_{pen1}	**Best**	0.00E+0 DE ≡	0.00E+0 μ-DE≡	0.00E+0 SADE≡	0.00E+0 EEv≡	0.00E+0 PSO≡	0.00E+0 μ-PSO
	IP	3.98E+1 EEv ≺	3.38E+2 μ-DE≺	5.81E+2 μ-PSO≺	6.33E+2 SADE≺	1.93E+3 PSO≺	4.78E+3 DE
	Speed	5.05E+2 EEv ≺	1.90E+3 μ-DE≺	2.18E+3 SADE≺	3.95E+3 μ-PSO≺	1.39E+4 PSO≺	2.51E+4 DE
	Success	1.0 DE ≡	1.0 μ-DE≡	1.0 SADE≡	1.0 EEv≡	1.0 PSO≡	1.0 μ-PSO
f_{pen2}	**Best**	0.00E+0 μ-PSO≡	0.00E+0 EEv ≡	0.00E+0 PSO≺	3.31E-3 μ-DE≺	1.70E-2 SADE≺	4.41E-2 DE
	IP	6.05E+1 EEv ≺	4.17E+2 μ-DE≺	7.05E+2 μ-PSO≺	7.69E+2 SADE≺	2.43E+3 PSO ≺	7.05E+3 DE
	Speed	3.69E+4 SADE≺	4.61E+4 μ-DE≺	4.77E+4 EEv≺	7.83E+4 μ-PSO≺	2.00E+5 DE ≺	2.24E+5 PSO
	Success	1.0 PSO≡	1.0 μ-PSO≡	1.0 EEv≻	0.92 SADE≻	0.90 DE ≻	0.22 μ-DE
f_{sal}	**Best**	1.98E-1 SADE≺	2.58E-1 DE ≺	5.07E-1 μ-PSO≺	5.30E-1 PSO≺	9.38E-1 EEv ≺	1.99E+0 μ-DE
	IP	6.72E+3 SADE≺	7.38E+3 EEv ≺	1.46E+4 μ-PSO≺	1.59E+4 μ-DE≺	4.50E+4 PSO≺	6.96E+4 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE≡	3.00E+5 SADE≡	3.00E+5 EEv≡	3.00E+5 PSO≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE≡	0.0 SADE≡	0.0 EEv≡	0.0 PSO≡	0.0 μ-PSO

Table 4. Multicriteria ranking of multimodal separable problems with D=30.

f_{sch}	**Best**	3.00E+1 μ-DE ≺	9.77E+1 SADE ≺	1.66E+2 DE ≺	1.45E+3 μ-PSO ≺	1.46E+3 PSO ≺	2.82E+3 EEv
	IP	5.24E+3 μ-DE ≺	1.84E+4 SADE ≺	8.73E+4 μ-PSO ≺	1.12E+5 DE ≺	1.54E+5 PSO ≺	1.66E+5 EEv
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{ras}	**Best**	4.48E-2 μ-DE ≺	4.31E+0 SADE ≺	5.71E+0 μ-PSO ≺	2.04E+1 EEv ≺	2.62E+1 DE ≺	2.78E+1 PSO
	IP	1.49E+4 μ-DE ≺	2.94E+4 SADE ≺	5.22E+4 EEv ≺	9.43E+4 μ-PSO ≺	1.06E+5 DE ≺	1.21E+5 PSO
	Speed	7.25E+4 μ-DE ≺	2.76E+5 SADE ≺	3.00E+5 DE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.12 μ-DE ≻	0.06 SADE ≻	0.0 DE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

Table 5. Multicriteria ranking of multimodal separable shifted problems with D=30.

f_{sch}	**Best**	3.10E+1 μ-DE ≺	1.04E+2 SADE ≺	1.83E+2 DE ≺	1.49E+3 μ-PSO ≺	1.52E+3 PSO ≺	2.85E+3 EEv
	IP	8.35E+3 μ-DE ≺	1.85E+4 SADE ≺	8.98E+4 μ-PSO ≺	1.10E+5 DE ≺	1.44E+5 PSO ≺	1.62E+5 EEv
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{ras}	**Best**	8.59E-2 μ-DE ≺	3.95E+0 SADE ≺	5.83E+0 μ-PSO ≺	2.05E+1 EEv ≺	2.53E+1 DE ≺	2.78E+1 PSO
	IP	1.86E+4 μ-DE ≺	2.85E+4 SADE ≺	5.43E+4 EEv ≺	9.46E+4 μ-PSO ≺	1.09E+5 DE ≺	1.18E+5 PSO
	Speed	2.87E+5 μ-DE ≡	2.88E+5 SADE ≺	3.00E+5 DE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.16 μ-DE ≻	0.01 SADE ≻	0.0 DE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

Table 6. Multicriteria ranking of unimodal shifted problems with D=30.

f_{sph}	**Best**	0.00E+0 DE ≡	0.00E+0 μ-PSO ≡	0.00E+0 SADE ≡	0.00E+0 EEv ≡	0.00E+0 PSO ≺	1.08E-2 μ-DE
	IP	1.17E+3 μ-DE ≺	1.41E+3 EEv ≺	1.47E+3 μ-PSO ≺	1.93E+3 SADE ≺	4.85E+3 PSO ≺	1.39E+4 DE
	Speed	1.89E+4 SADE ≺	5.05E+4 EEv ≺	8.48E+4 μ-PSO ≺	1.85E+5 DE ≺	2.25E+5 PSO ≺	2.41E+5 μ-DE
	Success	1.0 DE ≡	1.0 PSO ≡	1.0 SADE ≡	1.0 EEv ≡	1.0 μ-PSO ≻	0.30 μ-DE

Table 6. Cont'd.

$f_{2.22}$	**Best**	0.00E+0 SADE ≡	0.00E+0 μ-DE ≡	0.00E+0 μ-PSO ≡	0.00E+0 EEv ≡	0.00E+0 PSO <	9.63E-3 DE
	IP	2.27E+3 μ-DE <	2.44E+3 SADE <	4.55E+3 μ-PSO <	8.71E+3 PSO <	1.94E+4 DE <	9.25E+4 EEv
	Speed	5.24E+3 μ-DE ≡	6.16E+3 SADE <	1.11E+4 μ-PSO <	1.87E+4 PSO <	2.75E+4 DE <	1.46E+5 EEv
	Success	1.0 μ-DE ≡	1.0 PSO ≡	1.0 SADE ≡	1.0 EEv ≡	1.0 μ-PSO >	0.99 DE
$f_{2.21}$	**Best**	1.16E+1 SADE <	1.60E+1 EEv <	1.63E+1 DE ≡	1.64E+1 μ-PSO ≡	1.64E+1 PSO ≡	1.65E+1 μ-DE
	IP	3.05E+4 SADE <	4.35E+4 DE <	8.89E+4 μ-PSO <	9.75E+4 μ-DE <	1.36E+5 PSO <	2.27E+5 EEv
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{stp}	**Best**	0.00E+0 PSO ≡	0.00E+0 EEv ≡	0.00E+0 μ-PSO <	2.83E-2 SADE <	1.35E-1 DE <	1.92E-1 μ-DE
	IP	1.12E+3 μ-DE ≡	1.49E+3 μ-PSO <	1.97E+3 SADE <	2.64E+3 EEv <	4.98E+3 PSO <	1.39E+4 DE
	Speed	3.96E+4 SADE <	8.73E+4 μ-PSO <	9.50E+4 μ-DE ≡	1.13E+5 DE <	1.17E+5 PSO <	1.83E+5 EEv
	Success	1.0 μ-PSO ≡	1.0 PSO ≡	1.0 EEv >	0.97 SADE >	0.87 DE >	0.82 μ-DE
f_{qtc}	**Best**	0.00E+0 DE ≡	0.00E+0 μ-PSO ≡	0.00E+0 SADE ≡	0.00E+0 ≡EEv ≡	0.00E+0 PSO	8.22E-10 μ-DE
	IP	3.70E+2 μ-DE <	4.35E+2 EEv <	6.34E+2 μ-PSO <	1.02E+3 SADE <	2.05E+3 PSO <	3.99E+3 DE
	Speed	1.35E+4 SADE <	1.48E+4 EEv <	3.58E+4 μ-DE <	3.82E+4 μ-PSO <	8.30E+4 PSO <	9.33E+4 DE
	Success	1.0 DE ≡	1.0 PSO ≡	1.0 SADE ≡	1.0 EEv ≡	1.0 μ-PSO >	0.98 μ-DE
$f_{1.2}$	**Best**	9.58E-4 SADE <	1.62E-3 EEv <	9.05E-2 DE <	6.21E-1 μ-PSO <	1.61E+1 PSO <	5.29+4 μ-DE
	IP	2.00E+3 EEv <	4.88E+3 μ-PSO <	8.50E+3 SADE ≡	8.77E+3 μ-DE <	2.26E+4 PSO <	4.62E+4 DE
	Speed	2.89E+5 SADE <	3.00E+5 μ-DE ≡	3.00E+5 DE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.24 SADE >	0.0 μ-DE ≡	0.0 DE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

(30D, 30D shifted, 30D rotated, 100D and 200D). Then, "General μ-EAs behavior" Subsection presents a general discussion of the performances provided by the three μ-EAs. Finally, in "Comparison" against the simple adaptive differential evolution" Subsection, we comment on the performance of the μ-EAs with respect to that observed by a state-of-the-art approach (SADE). We say that an algorithm shows a competitive performance on a set of test problems in this paper, if it is the best or second best (out of six) by at least two criteria (out of four) in more than half of the test problems for such set. Criteria with artificial values indicating that the problem was not solved (e.g., 3.00E+5 for speed and 0.0 for success) were not considered. This method was adopted instead of the Pareto dominance criterion because the latter is not suitable for more than three objectives (four criteria in our

Table 7. Multicriteria ranking of multimodal non-separable shifted problems with D=30.

f_{ros}	**Best**	3.53E+1 DE <	5.32E+1 SADE <	5.73E+1 PSO <	6.60E+1 EEv <	7.27E+1 μ-DE <	9.70E+1 μ-PSO
	IP	3.66E+2 EEv <	4.84E+2 μ-DE <	5.39E+2 μ-PSO <	1.00E+3 SADE <	1.98E+3 PSO <	4.05E+3 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{ack}	**Best**	0.00E+0 SADE ≡	0.00E+0 EEv ≡	0.00E+0 DE ≡	0.00E+0 μ-PSO <	5.19E-5 PSO <	1.25E-3 μ-DE
	IP	4.34E+3 SADE <	4.80E+3 EEv <	6.53E+3 μ-PSO <	7.59E+3 μ-DE <	1.82E+4 PSO <	3.79E+4 DE
	Speed	2.82E+4 SADE <	1.04E+5 EEv <	2.65E+5 DE <	2.88E+5 μ-DE <	3.00E+5 μ-PSO ≡	3.00E+5 PSO
	Success	1.0 SADE ≡	1.0 μ-PSO ≡	1.0 DE ≡	1.0 EEv >	0.81 μ-DE >	0.0 PSO
f_{grw}	**Best**	1.77E-3 PSO <	3.62E-3 DE <	4.68E-3 SADE <	1.62E-2 μ-DE <	2.10E-2 μ-PSO <	2.44E-2 EEv
	IP	1.22E+3 μ-DE <	1.45E+3 EEv <	1.51E+3 μ-PSO <	1.95E+3 SADE <	4.99E+3 PSO <	1.40E+4 DE
	Speed	1.16E+5 SADE <	2.27E+5 DE <	2.45E+5 μ-PSO <	2.52E+5 EEv <	2.65E+5 PSO <	2.91E+5 μ-DE
	Success	0.83 SADE >	0.78 PSO >	0.71 DE >	0.27 μ-PSO >	0.23 EEv >	0.04 μ-DE
f_{pen1}	**Best**	0.00E+0 DE ≡	0.00E+0 μ-DE ≡	0.00E+0 SADE ≡	0.00E+0 EEv ≡	0.00E+0 PSO ≡	0.00E+0 μ-PSO
	IP	2.80E+2 μ-DE <	2.94E+2 EEv <	3.84E+2 μ-PSO <	7.75E+2 SADE <	1.34E+3 PSO <	2.73E+3 DE
	Speed	2.57E+3 EEv <	2.92E+3 μ-DE <	3.59E+3 SADE <	3.76E+3 μ-PSO <	1.22E+4 PSO <	2.73E+4 DE
	Success	1.0 DE ≡	1.0 μ-DE ≡	1.0 SADE ≡	1.0 EEv ≡	1.0 PSO ≡	1.0 μ-PSO
f_{pen2}	**Best**	7.68E-2 EEv <	7.77E-2 μ-PSO <	7.80E-2 PSO <	7.93E-2 DE <	8.41E-2 μ-DE <	1.11E-1 SADE
	IP	3.27E+2 μ-DE <	3.81E+2 EEv <	5.03E+2 μ-PSO <	9.00E+2 SADE <	1.74E+3 PSO <	3.16E+3 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{sal}	**Best**	1.98E-1 SADE <	2.60E-1 DE <	4.93E-1 μ-DE <	6.05E-1 PSO <	6.13E-1 μ-PSO <	1.03E+0 EEv
	IP	7.47E+3 SADE <	1.17E+4 EEv <	1.31E+4 μ-PSO <	2.19E+4 μ-DE <	3.59E+4 PSO <	6.44E+4 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

Table 8. Multicriteria ranking of unimodal rotated problems with D=30.

f_{sph}	**Best**	0.00E+0 DE ≡	0.00E+0 µ-PSO≡	0.00E+0 SADE ≡	0.00E+0 EEv <	2.24E-7 PSO<	3.98E-2 µ-DE
	IP	4.22E+2 EEv<	1.87E+3 SADE<	2.13E+3 µ-PSO<	2.58E+3 µ-DE <	7.67E+3 PSO<	1.74E+4 DE
	Speed	3.05E+4 SADE<	6.83E+4 µ-DE<	1.40E+5 EEv<	2.05E+5 µ-PSO<	2.72E+5 DE <	3.00E+5 PSO
	Success	1.0 DE≡	1.0 µ-PSO≡	1.0 SADE≡	1.0 EEv <	0.16 µ-DE<	0.0 PSO
$f_{2.22}$	**Best**	0.00E+0 DE≡	0.00E+0 SADE<	2.58E-1 EEv <	5.66E+0 PSO <	7.38E+0 µ-DE<	1.47E+1 µ-PSO
	IP	3.02E+3 µ-DE<	3.08E+4 SADE<	4.38+4 µ-PSO<	5.37E+4 PSO <	5.81E+4 DE <	7.03E+4 EEv
	Speed	4.61E+3 SADE<	4.33E+4 DE <	1.92E+5 EEv <	2.93E+5 PSO <	3.00E+5 µ-DE≡	3.00E+5 µ-PSO
	Success	1.0 DE≡	1.0 SADE>	0.84 EEv >	0.49 PSO >	0.0 µ-PSO≡	0.0 µ-DE
$f_{2.21}$	**Best**	2.14E+0 DE <	2.52E+0 SADE<	9.53E+0 EEv<	3.16E+1 PSO<	3.69E+1 µ-DE<	4.86E+1 µ-PSO
	IP	7.94E+3 SADE<	6.77E+4 DE <	7.61E+4 EEv<	7.95E+4 µ-PSO<	8.14E+4 µ-DE<	1.69E+5 PSO
	Speed	3.00E+5 DE≡	3.00E+5 µ-DE≡	3.00E+5 SADE≡	3.00E+5 EEv ≡	3.00E+5 PSO≡	3.00E+5 µ-PSO
	Success	0.0 DE≡	0.0 µ-DE≡	0.0 SADE≡	0.0 EEv ≡	0.0 PSO≡	0.0 µ-PSO
f_{stp}	**Best**	5.10E-2 DE<	3.92E-1 SADE<	7.54E+0 µ-DE<	8.31E+0 PSO <	1.02E+1 EEv<	1.38E+1 µ-PSO
	IP	8.16E+2 EEv<	1.87E+3 SADE<	2.16E+3 µ-PSO<	2.79E+3 µ-DE <	7.43E+3 PSO<	1.79E+4 DE
	Speed	1.00E+5 DE <	1.13E+5 SADE<	3.00E+5 µ-DE≡	3.00E+5 EEv ≡	3.00E+5 PSO≡	3.00E+5 µ-PSO
	Success	0.95 DE >	0.72 SADE>	0.0 µ-DE≡	0.0 µ-PSO≡	0.0 PSO≡	0.0 EEv
f_{qtc}	**Best**	0.00E+0 DE≡	0.00E+0 µ-PSO≡	0.00E+0 SADE≡	0.00E+0 EEv ≡	0.00E+0 PSO<	1.33E-9 µ-DE
	IP	5.22E+1 EEv<	6.46E+2 µ-PSO<	6.87E+2 µ-DE<	1.00E+3 SADE<	2.69E+3 PSO<	9.78E+3 DE
	Speed	1.60E+4 SADE<	1.85E+4 EEv<	4.40E+4 µ-PSO<	9.48E+4 µ-DE <	1.02E+5 DE <	1.19E+5 PSO
	Success	1.0 DE≡	1.0 µ-PSO≡	1.0 SADE≡	1.0 EEv ≡	1.0 PSO>	0.98 µ-DE
$f_{1.2}$	**Best**	5.46E-2 EEv<	4.35E-1 DE <	1.55E+0 µ-PSO<	1.91E+1 SADE<	7.80E+1 PSO<	8.56E+3 µ-DE
	IP	5.82E+2 EEv<	4.72E+3 µ-PSO<	7.93E+3 SADE<	1.67E+4 µ-DE<	2.87E+4 PSO<	4.79E+4 DE
	Speed	3.00E+5 DE≡	3.00E+5 µ-DE≡	3.00E+5 SADE≡	3.00E+5 EEv ≡	3.00E+5 PSO≡	3.00E+5 µ-PSO
	Success	0.01 SADE>	0.0 µ-DE≡	0.0 DE ≡	0.0 EEv ≡	0.0 PSO≡	0.0 µ-PSO

Table 9. Multicriteria ranking of multimodal non-separable rotated problems with D=30.

f_{ros}	**Best**	2.54E+1 PSO <	4.48E+1 DE <	5.13E+1 SADE <	6.66E+1 EEv <	1.07E+2 μ-PSO <	8.10E+2 μ-DE
	IP	7.60E+1 EEv <	6.73E+2 μ-PSO ≡	6.74E+2 μ-DE <	9.44E+2 SADE <	2.53E+3 PSO <	9.86E+3 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{ack}	**Best**	0.00E+0 SADE ≡	0.00E+0 EEv <	2.68E-9 DE <	7.44E-9 μ-PSO <	1.83E-5 PSO <	3.62E-3 μ-DE
	IP	3.10E+3 EEv <	4.54E+3 SADE <	8.43E+3 μ-PSO <	1.41E+4 μ-DE <	2.68E+4 PSO <	4.34E+4 DE
	Speed	4.51E+4 SADE <	1.33E+5 EEv <	2.87E+5 μ-DE <	2.88E+5 DE <	2.93E+5 μ-PSO <	3.00E+5 PSO
	Success	1.0 SADE ≡	1.0 EEv >	0.86 DE >	0.50 μ-PSO >	0.07 μ-DE >	0.0 PSO
f_{grw}	**Best**	1.22E-3 DE <	1.45E-3 PSO <	7.85E-3 SADE ≡	8.16E-3 μ-PSO <	1.70E-2 EEv <	1.01E-1 μ-DE
	IP	4.45E+2 EEv <	1.89E+3 SADE <	2.19E+3 μ-PSO <	2.78E+3 μ-DE <	7.54E+3 PSO <	1.74E+4 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.01 μ-PSO >	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 DE
f_{pen1}	**Best**	0.00E+0 DE ≡	0.00E+0 μ-DE ≡	0.00E+0 SADE ≡	0.00E+0 EEv ≡	0.00E+0 PSO ≡	0.00E+0 μ-PSO
	IP	4.68E+1 EEv <	4.36E+2 μ-PSO <	5.32E+2 μ-DE <	6.87E+2 SADE <	1.84E+3 PSO <	5.76E+3 DE
	Speed	1.55E+3 EEv <	2.92E+3 SADE <	8.20E+3 μ-PSO <	1.95E+4 PSO <	2.38E+4 μ-DE <	2.97E+4 DE
	Success	1.0 DE ≡	1.0 μ-DE ≡	1.0 SADE ≡	1.0 EEv ≡	1.0 PSO ≡	1.0 μ-PSO
f_{pen2}	**Best**	8.79E-4 DE <	3.20E-3 SADE <	7.71E-1 μ-DE <	2.49E+1 PSO <	3.47E+1 EEv <	5.84E+1 μ-PSO
	IP	6.71E+1 EEv <	5.62E+2 μ-DE <	5.87E+2 μ-PSO <	7.97E+2 SADE <	2.22E+3 PSO <	6.78E+3 DE
	Speed	1.13E+5 SADE <	2.39E+5 DE <	2.99E+5 EEv <	3.00E+5 μ-DE ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.94 DE ≡	0.75 SADE ≡	0.01 EEv >	0.0 μ-PSO ≡	0.0 PSO ≡	0.0 μ-DE
f_{sal}	**Best**	2.19E-1 SADE <	2.72E-1 DE <	5.65E-1 μ-DE <	6.96E-1 μ-PSO <	7.19E-1 PSO <	1.17E+0 EEv
	IP	6.55E+3 SADE <	6.79E+3 EEv <	1.73E+4 μ-PSO <	3.68E+4 μ-DE <	5.11E+4 PSO <	6.34E+4 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

Table 10. Multicriteria ranking of multimodal separable rotated problems with D=30.

f_{ras}	**Best**	5.35E+1 SADE<	1.04E+2 DE <	1.08E+2 PSO<	1.12E+2 EEv <	1.62E+2 μ-DE<	1.89E+2 μ-PSO
	IP	2.46E+3 EEv <	1.41E+4 μ-PSO<	4.92E+4 PSO<	1.23E+5 μ-DE<	1.44E+5 SADE<	1.96E+5 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE≡	3.00E+5 SADE≡	3.00E+5 EEv ≡	3.00E+5 PSO≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE≡	0.0 SADE≡	0.0 EEv ≡	0.0 PSO≡	0.0 μ-PSO

Table 11. Multicriteria ranking of multimodal separable problems with D=100.

f_{sch}	**Best**	1.24E+3 μ-DE<	2.30E+3 SADE<	8.17E+3 μ-PSO<	9.19E+3 DE <	1.03E+4 PSO<	1.92E+4 EEv
	IP	5.16E+4 μ-DE<	6.92E+4 SADE<	1.49E+5 EEv <	1.82E+5 μ-PSO≡	1.82E+5 PSO<	2.61E+5 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE≡	3.00E+5 EEv≡	3.00E+5 PSO≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE≡	0.0 EEv ≡	0.0 PSO≡	0.0 μ-PSO
f_{ras}	**Best**	4.47E+1 SADE<	1.72E+2 μ-PSO<	1.80E+2 DE <	1.83E+2 EEv<	2.52E+2 μ-DE<	3.05E+2 PSO
	IP	9.20E+3 EEv <	6.21E+4 SADE <	1.25E+5 PSO<	1.62E+5 μ-PSO<	1.76E+5 μ-DE<	1.98E+5 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE≡	3.00E+5 EEv≡	3.00E+5 PSO≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE≡	0.0 EEv ≡	0.0 PSO≡	0.0 μ-PSO

Table 12. Multicriteria ranking of unimodal problems with D=100.

f_{sph}	**Best**	0.00E+0 SADE≡	0.00E+0 EEv<	4.11E-7 μ-PSO<	1.49E-1 PSO<	1.11E+1 DE <	6.57E+1 μ-DE
	IP	8.05E+2 EEv <	2.88E+3 SADE<	7.41E+3 μ-PSO<	2.18E+4 μ-DE<	3.20E+4 PSO<	4.40E+4 DE
	Speed	8.78E+4 SADE<	2.01E+5 EEv<	3.00E+5 DE ≡	3.00E+5 μ-PSO≡	3.00E+5 μ-DE≡	3.00E+5 PSO
	Success	1.0 SADE≡	1.0 EEv>	0.0 DE ≡	0.0 μ-DE≡	0.0 PSO≡	0.0 μ-PSO
$f_{2.22}$	**Best**	0.00E+0 EEv≡	0.00E+0 μ-PSO<	1.39E-1 PSO <	8.21E-1 μ-DE<	1.92E+0 SADE<	7.10E+0 DE
	IP	3.94E+3 SADE<	2.36E+4 μ-PSO<	2.78E+4 μ-DE <	5.13E+4 DE<	6.81E+4 PSO <	9.34E+4 EEv
	Speed	8.60E+4 μ-PSO<	1.74E+5 EEv<	2.09E+5 μ-DE ≡	2.13E+5 PSO<	2.49E+5 SADE<	3.00E+5 DE
	Success	1.0 μ-PSO≡	1.0 EEv>	0.87 PSO >	0.42 μ-DE>	0.22 SADE>	0.0 DE
$f_{2.21}$	**Best**	2.47E+1 SADE<	3.17E+1 EEv<	4.86E+1 DE <	6.19E+1 μ-DE<	6.33E+1 PSO<	6.55E+1 μ-PSO

Table 12. Cont'd.

	IP	1.39E+4 SADE <	8.40E+4 DE <	1.45E+5 μ-PSO <	1.67E+5 PSO <	1.76E+5 μ-DE <	1.97E+5 EEv
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{stp}	**Best**	3.41E-1 μ-PSO <	1.03E+1 PSO <	2.12E+1 EEv <	1.33E+2 μ-DE ≡	1.34E+2 SADE <	2.61E+2 DE
	IP	1.15E+3 EEv <	2.94E+3 SADE <	7.44E+3 μ-PSO <	2.17E+4 μ-DE <	3.24E+4 PSO <	4.42E+4 DE
	Speed	2.41E+5 μ-PSO <	3.00E+5 SADE ≡	3.00E+5 μ-DE ≡	3.00E+5 DE ≡	3.00E+5 PSO ≡	3.00E+5 EEv
	Success	0.25 μ-PSO >	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 DE
f_{qtc}	**Best**	0.00E+0 EEv ≡	0.00E+0 μ-PSO ≡	0.00E+0 SADE <	1.32E-6 PSO <	4.17E-3 DE <	1.04E-2 μ-DE
	IP	6.81E+1 EEv <	1.48E+3 SADE <	3.15E+3 μ-PSO <	4.98E+3 μ-DE <	1.57E+4 PSO <	1.97E+4 DE
	Speed	4.91E+4 EEv <	6.56E+4 SADE <	1.04E+5 μ-PSO <	2.97E+5 μ-DE <	3.00E+5 PSO ≡	3.00E+5 DE
	Success	1.0 μ-PSO ≡	1.0 EEv ≡	1.0 SADE >	0.02 μ-DE >	0.0 PSO ≡	0.0 DE
$f_{1.2}$	**Best**	6.36E+2 EEv <	6.91E+3 SADE <	7.59E+3 μ-PSO <	4.30E+4 PSO <	1.11E+5 μ-DE <	2.49E+5 DE
	IP	6.89E+2 EEv <	1.46E+4 μ-DE <	1.70E+4 μ-PSO ≡	2.11E+4 SADE <	6.91E+4 DE <	7.50E+4 PSO
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

Table 13. Multicriteria ranking of multimodal non-separable problems with D=100.

f_{ros}	**Best**	1.54E+2 EEv <	2.03E+2 SADE <	2.24E+2 μ-PSO <	5.04E+2 PSO <	4.05E+3 DE <	9.65E+3 μ-DE
	IP	1.05E+2 EEv <	1.55E+3 SADE <	3.69E+3 μ-PSO <	6.45E+3 μ-DE <	1.75E+4 PSO <	2.11E+4 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{ack}	**Best**	0.00E+0 SADE <	3.24E-8 EEv <	7.38E-6 μ-PSO <	4.96E-3 PSO <	3.07E-2 DE <	7.66E-2 μ-DE
	IP	8.16E+3 EEv <	1.17E+4 SADE <	2.23E+4 μ-PSO <	5.85E+4 μ-DE <	8.99E+4 PSO <	1.43E+5 DE
	Speed	1.17E+5 SADE <	3.00E+5 μ-PSO ≡	3.00E+5 μ-DE ≡	3.00E+5 DE ≡	3.00E+5 PSO ≡	3.00E+5 EEv
	Success	1.0 SADE >	0.0 μ-DE ≡	0.0 DE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

Table 13. Cont'd.

f_{grw}	**Best**	3.91E-3 μ-PSO <	5.27E-3 EEv <	6.39E-2 SADE <	8.61E-2 PSO <	8.98E-2 DE <	1.28E+0 μ-DE
	IP	8.76E+2 EEv <	2.89E+3 SADE <	7.41E+3 μ-PSO <	2.18E+4 μ-DE <	3.25E+4 PSO <	4.41E+4 DE
	Speed	2.08E+5 SADE <	2.77E+5 EEv <	3.00E+5 μ-DE ≡	3.00E+5 DE ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.54 SADE >	0.49 EEv ≡	0.0 DE ≡	0.0 μ-DE ≡	0.0 PSO ≡	0.0 μ-PSO
$f_{pen\,1}$	**Best**	0.00E+0 μ-PSO ≡	0.00E+0 μ-DE ≡	0.00E+0 SADE ≡	0.00E+0 EEv ≡	0.00E+0 PSO <	2.60E+4 DE
	IP	4.54E+1 EEv <	1.09E+3 SADE <	2.35E+3 μ-DE ≡	2.35E+3 μ-PSO <	1.11E+4 PSO <	1.49E+4 DE
	Speed	2.11E+3 EEv <	1.25E+4 SADE <	2.04E+4 μ-PSO <	6.55E+4 μ-DE <	1.14E+5 PSO <	2.81E+5 DE
	Success	1.0 μ-PSO ≡	1.0 μ-DE ≡	1.0 SADE ≡	1.0 EEv ≡	1.0 PSO >	0.26 DE
$f_{pen\,2}$	**Best**	4.10E-8 EEv <	1.47E-5 μ-PSO <	6.26E-1 SADE <	2.15E-1 PSO <	3.37E+3 μ-DE <	8.65E+4 DE
	IP	6.74E+1 EEv <	1.31e+3 SADE <	3.03E+3 μ-PSO <	3.84E+3 μ-DE <	1.43E+4 PSO <	1.75E+4 DE
	Speed	2.03E+5 SADE <	2.54E+5 EEv <	3.00E+5 μ-DE ≡	3.00E+5 DE ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.92 EEv >	0.68 SADE >	0.0 μ-DE ≡	0.0 μ-PSO ≡	0.0 PSO ≡	0.0 DE
f_{sal}	**Best**	1.08E+0 SADE <	2.15E+0 μ-PSO <	2.84E+0 EEv <	3.09E+0 μ-DE <	3.61E+0 PSO <	4.64E+0 DE
	IP	2.90E+4 SADE <	3.51E+4 EEv <	5.91E+4 μ-PSO <	1.57E+5 μ-DE <	1.60E+5 DE <	1.75E+5 PSO
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

case).

Performance analysis of each μ-EA

The detailed discussion for each compared μ-EA and SADE is presented here and subsequently. Furthermore, a summary of the findings can be found in Table 17.

Elitist evolution: EEv was competitive on five of six 30D unimodal problems (Table 2) and on five of six 30D multimodal non-separable problems (Table 3). However, it was not competitive on 30D multimodal separable test problems (Table 4).

Regarding 30D shifted test problems EEv was competitive on five of six shifted unimodal 30D test problems (Table 6). Nonetheless, EEv was not competitive on shifted 30D multimodal separable test problems (Table 5) and on 30D shifted multimodal non-separable test problems (Table 7).

The results on 30D rotated functions indicated that EEv was competitive on four of six 30D rotated unimodal test problems (Table 8) while on 30D rotated multimodal non-separable test problems (Table 9) and on 30D rotated multimodal separable test problems (Table 10) it did not provide a competitive performance.

On 100D test problems EEv was competitive on four of six 100D unimodal test problems and on five of six 100D multimodal non-separable test problems (Tables 12 and 13, respectively). In contrast, EEv was not competitive on 100D shifted multimodal separable test problems (Table 11).

Finally, on 200D test problems EEv provided a competitive performance on four of six 200D unimodal test problems (Table 14) and on five of six 200D

Table 14. Multicriteria ranking of unimodal problems with D=200.

f_{sph}	**Best**	2.52E-6 EEv <	2.49E-5 SADE <	6.36E-3 μ-PSO <	1.57E+2 PSO <	7.20E+2 μ-DE <	4.69E+3 DE
	IP	1.09E+3 EEv <	4.61E+3 SADE <	1.61E+4 μ-PSO <	7.55E+4 PSO <	8.78E+4 DE <	9.26E+4 μ-DE
	Speed	2.54E+5 SADE <	3.00E+5 μ-DE ≡	3.00E+5 EEv ≡	3.00E+5 μ-PSO ≡	3.00E+5 DE ≡	3.00E+5 PSO
	Success	0.66 SADE >	0.0 μ-DE ≡	0.0 DE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
$f_{2.22}$	**Best**	4.76E-1 μ-PSO <	5.61E+0 μ-DE <	1.29E+1 EEv <	1.94E+1 PSO <	2.28E+1 SADE <	5.26E+1 DE
	IP	6.10E+3 SADE <	5.80E+4 μ-PSO <	8.02E+4 DE <	9.84E+4 μ-DE <	1.54E+5 PSO <	1.57E+5 EEv
	Speed	2.45E+5 μ-PSO <	2.99E+5 μ-DE <	2.99E+5 EEv <	3.00E+5 SADE ≡	3.00E+5 PSO ≡	3.00E+5 DE
	Success	0.69 μ-PSO >	0.06 EEv >	0.02 μ-DE >	0.0 PSO ≡	0.0 SADE ≡	0.0 DE
$f_{2.21}$	**Best**	3.27E+1 SADE <	4.56E+1 EEv <	6.22E+1 DE <	7.71E+1 PSO <	7.76E+1 μ-PSO <	8.37E+1 μ-DE
	IP	1.28E+4 SADE <	1.24E+5 DE <	1.35E+5 μ-PSO <	1.62E+5 μ-DE <	1.72E+5 PSO <	1.95E+5 EEv
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{stp}	**Best**	8.33E+0 μ-PSO <	3.25E+2 EEv <	4.63E+2 PSO <	1.05E+3 μ-DE <	1.58E+3 SADE <	6.04E+3 DE
	IP	1.23E+3 EEv <	4.46E+3 SADE <	1.68E+4 μ-PSO <	7.69E+4 PSO <	8.45E+4 DE <	9.09E+4 μ-DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{qtc}	**Best**	0.00E+0 EEv ≡	0.00E+0 μ-PSO <	1.45E-7 SADE <	9.10E-2 PSO <	2.69E+0 μ-DE <	1.03E+1 DE
	IP	6.93E+1 EEv <	1.72E+3 SADE <	6.99E+3 μ-PSO <	2.87E+4 DE <	3.71E+4 μ-DE <	3.74E+4 PSO
	Speed	1.17E+5 EEv <	1.97E+5 SADE <	2.56E+5 μ-PSO <	3.00E+5 μ-DE <	3.00E+5 PSO <	3.00E+5 DE
	Success	1.0 EEv ≡	1.0 μ-PSO >	0.95 SADE >	0.0 μ-DE ≡	0.0 DE ≡	0.0 PSO
$f_{1.2}$	**Best**	1.38E+4 EEv <	6.05E+4 SADE <	9.66E+4 μ-PSO <	2.51E+4 PSO <	5.31E+5 μ-DE <	1.03E+6 DE
	IP	6.66E+2 EEv <	9.64E+3 μ-DE <	1.59E+4 μ-PSO <	2.37E+4 SADE <	6.50E+4 DE ≡	6.66E+4 PSO
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

multimodal non-separable test problems (Table 15), but it was not the case on the 200D multimodal separable test

Table 15. Multicriteria ranking of multimodal non-separable problems with D=200.

f_{ros}	**Best**	3.42E+2 EEv <	6.44E+2 SADE<	7.15E+2 μ-PSO<	5.08E+4 PSO <	1.50E+6 μ-DE <	3.69E+6 DE
	IP	1.07E+2 EEv <	1.76E+3 SADE<	7.98E+3 μ-PSO<	3.07E+4 DE <	4.28E+4 PSO <	5.62E+4 μ-DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{ack}	**Best**	8.71E-6 SADE<	1.04E-5 EEv <	4.98E-4 μ-PSO <	7.93E-2 PSO <	1.62E-1 μ-DE <	4.27E-1 DE
	IP	1.63E+4 EEv <	3.21E+4 SADE<	4.78E+4 μ-PSO<	1.77E+5 PSO <	1.87E+5 μ-DE <	2.06E+5 DE
	Speed	2.86E+5 SADE<	3.00E+5 μ-DE≡	3.00E+5 DE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.32 SADE>	0.0 μ-DE≡	0.0 DE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{grw}	**Best**	2.29E-3 EEv <	7.59E-3 μ-PSO<	2.01E-1 SADE<	2.39E+0 PSO<	7.48E+0 μ-DE<	4.32E+1 DE
	IP	1.27E+3 EEv <	4.68E+3 SADE<	1.59E+4 μ-PSO<	7.48E+4 PSO <	8.78E+4 DE <	9.26E+4 μ-DE
	Speed	2.87E+5 SADE<	3.00E+5 μ-DE≡	3.00E+5 DE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.28 SADE>	0.0 μ-DE≡	0.0 DE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{pen1}	**Best**	0.00E+0 EEv≡	0.00E+0 μ-PSO<	6.42E+2 PSO <	3.13E+3 SADE<	2.44E+6 μ-DE<	2.60E+6 DE
	IP	4.43E+1 EEv<	1.35E+3 SADE<	5.42E+3 μ-PSO <	1.73E+4 μ-DE≡	1.93E+4 DE ≡	2.98E+4 PSO
	Speed	4.35E+3 EEv<	5.18E+4 μ-PSO<	1.03E+5 SADE <	3.00E+5 μ-DE≡	3.00E+5 DE ≡	3.00E+5 PSO
	Success	1.0 EEv≡	1.0 μ-PSO>	0.97 SADE >	0.03 PSO >	0.0 DE ≡	0.0 μ-DE
f_{pen2}	**Best**	5.74E-1 μ-PSO<	1.20E+2 EEv <	3.34E+2 SADE<	1.81E+4 PSO <	5.47E+6 μ-DE <	9.55E+6 DE
	IP	7.21E+1 EEv <	1.57E+3 SADE<	6.63E+3 μ-PSO<	2.39E+4 DE <	3.34E+4 μ-DE <	3.63E+4 PSO
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{sal}	**Best**	4.16E+0 SADE<	5.21E+0 μ-PSO<	5.96E+0 EEv <	1.42E+1 PSO <	1.55E+1 DE <	1.70E+1 μ-DE
	IP	6.68E+4 EEv <	6.77E+4 SADE<	1.22E+5 μ-PSO <	1.79E+5 DE <	2.32E+5 PSO <	2.39E+5 μ-DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

1. The performance of μ-EAs is low on rotated problems. Rotation operator causes a stronger relation among the

Table 16. Multicriteria ranking of multimodal separable problems with D=200.

f_{sch}	**Best**	4.88E+3 μ-DE <	8.67E+3 SADE <	2.23E+4 μ-PSO <	2.84E+4 PSO <	3.60E+4 DE <	4.37E+4 EEv
	IP	4.42E+4 EEv <	1.24E+5 SADE <	1.90E+5 μ-PSO <	1.99E+5 μ-DE <	2.27E+5 PSO <	2.81E+5 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO
f_{ras}	**Best**	1.39E+2 SADE <	3.75E+2 EEv <	5.81E+2 DE <	6.13E+2 μ-PSO <	8.29E+2 PSO <	1.26E+3 μ-DE
	IP	1.77E+4 EEv <	8.47E+4 SADE <	1.53E+5 μ-PSO ≡	1.54E+5 μ-DE <	1.78E+5 PSO <	2.30E+5 DE
	Speed	3.00E+5 DE ≡	3.00E+5 μ-DE ≡	3.00E+5 SADE ≡	3.00E+5 EEv ≡	3.00E+5 PSO ≡	3.00E+5 μ-PSO
	Success	0.0 DE ≡	0.0 μ-DE ≡	0.0 SADE ≡	0.0 EEv ≡	0.0 PSO ≡	0.0 μ-PSO

Table 17. Summary of competitive performances (✓) by EEv, the two compared μ-EAs and SADE.

Set of problem	EEV	μ-PSO	μ-DE	SADE
30D unimodal	✓	✓	✓	✓
30D multimodal non-separable	✓	O	O	✓
30D multimodal separable	O	O	✓	✓
30D shifted unimodal	✓	✓	O	✓
30D shifted multimodal non-separable	O	O	O	✓
30D shifted multimodal separable	O	O	✓	✓
30D rotated unimodal	✓	O	O	✓
30D rotated multimodal non-separable	O	O	O	✓
30D rotated multimodal separable	O	O	O	O
100D unimodal	✓	O	O	✓
100D multimodal non-separable	✓	O	O	✓
100D multimodal separable	O	O	O	✓
200D unimodal	✓	O	O	O
200D multimodal non-separable	✓	O	O	✓
200D multimodal separable	O	O	O	✓

An “O” means that the algorithm was not competitive in such problem set.

dimensions of the problem, which increases the effect of a small change in one dimension. An exception is EEv on 30D rotated unimodal problems.

2. Both, EEv and μ-PSO provided a competitive performance on shifted 30D unimodal problems, while μ-DE was able to be competitive on shifted 30D multimodal separable problems. Shifted 30D non-separable multimodal problems were difficult to solve for all three μ-EAs.
3. Rotation operator seems to affect the performance of μ-EAs more than that of the original EAs.
4. EEv was the only competitive μ-EA on 100D and 200D test problems (only in those with unimodal and multimodal non-separable functions).
5. μ-DE was the only competitive μ-EA on 30D shifted or non-shifted multimodal separable test problems.
6. μ-EAs over performed EAs (DE/rand/1/bin and local-best PSO) in terms of success rate, speed and precision in 9 out of 14 problems.
7. μ-EAs improvement periods are always smaller than those observed for their EAs counterparts. μ-EAs seem to work faster.

Comparison against the simple adaptive differential evolution

Based on the aforementioned comparison, EEv was the most competitive μ-EA. Here we compare its results against SADE as one of the most competitive algorithms found in the specialized literature on global optimization with evolutionary algorithms (Qin et al., 2009).

From the results in Tables 2 to 16, it can be observed that SADE was competitive on all six 30D unimodal problems (Table 2), on five of six 30D multimodal non-separable problems (Table 3), and on the two 30D multimodal separable test problems (Table 4). Moreover, SADE was competitive on the two 30D shifted multimodal separable test problems (Table 5), on the six 30D shifted unimodal test problems (Table 6), and on four of six 30D shifted multimodal non-separable test problems (Table 7). Furthermore, on the 30D rotated functions SADE was competitive on five of six 30D rotated unimodal test problems (Table 8), on four of six 30D rotated multimodal non-separable test problems (Table 9), but it was not the case on the 30D rotated multimodal separable test problems (Table 10).

On those 100D test problems, SADE was competitive on the two 100D shifted multimodal separable test problems (Table 11), on four of six 100D unimodal test problems (Table 12), and on all six 100D multimodal non-separable test problems (Table 13).

Finally, on the 200D test problems, SADE was not competitive in 200D unimodal problems (Table 14). However, it was competitive on five of six 200D multimodal non-separable problems (Table 15) and on the two 200D multimodal separable problems (Table 16).

From the overall results shown by EEv and SADE, it is clear that the latter is the most competitive algorithm according to the comparison criteria adopted in this paper. However, EEv remained very competitive in 30D rotated unimodal test problems, 100D unimodal and 100D multimodal non-separable test problems. Furthermore, EEv was more competitive on 200D unimodal test problems.

The main feature both, EEv and SADE share is that they have adaptive mechanisms which seem to be one of the reasons why they outperform the other algorithms.

Finally, by contrasting the visual graphical comparison presented previously with these findings, it is clear that the behavior on higher dimensional problems is almost the opposite to that observed on the 2D problem, that is, the algorithms with adaptive mechanisms provide better results with lower FEs values and those PSO-based algorithms presented premature convergence.

It is well-known from the no free lunch theorems for search (Wolpert and Macready, 1997) that using a limited set of functions cannot guarantee that an algorithm providing a competitive performance in them will do the same in a different set of problems. However, the aim of this analysis is to obtain some knowledge on the behavior of EEv and other μ-EAs and EAs in different types of search spaces.

CONCLUSIONS AND FUTURE WORK

An empirical analysis of EEv, a μ-EA designed to work with small populations, was presented in this paper. EEv was used to solve a wide set of test problems with different features such as unimodal, multimodal, shifted and rotated functions, besides using different dimensionalities. Different μ-EAs and other EAs plus one state-of-the-art algorithm were used in the comparison of results. A very general graphical comparison on a well-known test problem in 2D showed that EEv shared with SADE (two EAs with adaptive mechanisms) a higher number of FEs to converge. On the other hand, the empirical comparison, validated with a recently proposed statistical methodology, showed that EEv was the most competitive μ-EA with respect to μ-PSO and μ-DE, while its performance was almost comparable with respect to that observed by SADE, a state-of-the-art algorithm for global optimization. It is worth remarking that EEv provided better results in problems with a high dimensionality with respect to the other two μ-EAs. On the other hand, EEv's performance was affected by some rotated and shifted functions. Nonetheless, EEv was able to provide competitive results in 30D shifted unimodal and 30D rotated unimodal functions. The overall results presented in this work suggest that EEv, which has simple mutation and crossover operators coupled with a combination of elitism and adaptive behavior, is able to provide competitive results even in complex search spaces.

Part of the future work includes a comprehensive analysis of the parameter values used in EEv and their relationship with sources of difficulty found in this work such as shifted and rotated functions and also the solution of real-world problems, mostly related with mechanical design.

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REFERENCES

Carrano EG, Wanner EF, Takahashi RHC (2011). A Multicriteria Statistical Based Comparison Methodology for Evaluating Evolutionary Algorithms. IEEE Trans. Evol. Comput., (99): 1-23.

Deb K, Anand A, Joshi D (2002). A computationally efficient evolutionary algorithm for real-parameter optimization.Evol. Comput., 10(4): 371-395.

Eiben AE, Smith JE (2003). Introduction to Evolutionary Computing. Springer.

Fuentes-Cabrera JC, Coello-Coello CA (2007). Handling Constraints in Particle Swarm Optimization using a Small Population Size. Proc. Mex. Int. Conf.Artif.Intell.,Lect. Notes Artif. Intell., 4827: 41-51.

Goldberg DE (1989). Sizing Populations for Serial and Parallel Genetic Algorithms. Proc. Int.Conf.Genet. Algorithms, pp. 70-79.

Kattan A, Poli R (2008). Evolutionary lossless compression with GP-ZIP*. Proc.Genetic.Evol. Comput. Conf., pp. 1211-1218.

Kazarlis SE, Papadakis SE, Theocharis JB, Petridis V (2001). Microgenetic Algorithms as Generalized Hill-Climbing Operators for GA Optimization. IEEE Trans. Evol. Comput., 5(3): 204-217.

Kleeman MP, Lamont GB (2005). Solving the Aircraft Engine Maintenance Scheduling Problem Using a Multi-objective Evolutionary Algorithm. Proc.Evolut. Multi-Criterion Optim. Conf., Lect. Notes Comput. Sci., 3410: 782-796.

Krishnakumar K (1989). Micro-genetic algorithms for stationary and non-stationary function optimization.SPIE: Intell. Cont. Adapt. Syst., 1196: 289-296.

Hassan FR, Koh SP, Tiong SK, Chong KH, Abdalla AN (2011). Investigation of induction motor parameter identification using particle swarm optimization-based RBF neural network (PSO-RBFNN). Int. J. Phys. Sci., 6(9): 4564-4570.

Mezura-Montes E, Coello-Coello CA, Velazquez-Reyes J (2006). A comparative study of differential evolution variants for global optimization.Proc.Genetic.Evol.Comput. Conf., pp. 485-492.

Noman N, Iba H (2008). Accelerating Differential Evolution Using an Adaptive Local Search,IEEE Trans. Evol. Comput., 12(1): 107-125.

Noor MM, Kadirgama K, Rahman MM (2011). Particle swarm optimisation prediction model for surface roughness. Int. J. Phys. Sci., 6(13): 3082-3090.

Parsopoulos KE (2009). Cooperative micro-differential evolution for high-dimensional problems. Proc. Genetic Evol. Comput. Conf., pp. 531-538.

Qin KA, Huang VL, uganthan PN (2009). Differential Evolution Algorithm With Strategy Adaptation for Global Numerical Optimization,IEEE Trans. Evol. Comput., 13(2): 398-417.

Satoh H, Yamamura M, Kobayashi S (1996). Minimal generation gap models for GAs considering both exploration and exploitation. Proc. Int. Conf. Fuzzy Logic Neural Nets Soft Comput., pp. 494-497.

Storn R, Price KV (1997). Differential Evolution -- a simple and efficient heuristic for global optimization. J. Glob. Optim., 11(4): 341-359.

Toscano-Pulido G, Coello-Coello CA (2001).Multiobjective Optimization using a Micro-Genetic Algorithm. Proc.Genetic.Evol.Comput. Conf., pp. 126-140.

Valsalam VK, Miikkulainen R (2008). Modular neuroevolution for multilegged locomotion. Proc. Genetic. Evol. Comput. Conf., pp. 265-272.

Viveros-Jiménez F, Mezura-Montes E, Gelbukh A (2009). Elitistic Evolution: An Efficient Heuristic for Global Optimization. Proc. Int. Conf. Adapt. Nat. Comput. Algorithms, Lect. Notes Comput. Sci., 5495: 171-182.

Wolpert DH, Macready WG (1997). No free lunch theorems for optimization. IEEE Trans. Evol. Comput., 1(1): 67-82.

Yan W, Sewell MV, Clack CD (2009). Learning to optimize profits beats predicting returns -: comparing techniques for financial portfolio optimization. Proc.Genetic.Evol.Comput. Conf., pp. 1681-1688.

APPENDIX

Test functions (Mezura-Montes et al., 2006; Noman et al., 2008):

f_{sph} - **Sphere model** $f_{sph}(x) = \sum_{i=1}^{D} x_i^2$ $-100 \leq x_i \leq 100$ $\min(f(x^*)) = f_{sph}(0, \dots, 0) = 0$	$f_{2.22}$ – **Schwefel's problem 2.22** $f_{2.22}(x) = \sum_{i=1}^{D} \lvert x_i \rvert + \prod_{i=1}^{D} \lvert x_i \rvert$ $-10 \leq x_i \leq 10$ $\min(f(x^*)) = f_{2.22}(0, \dots, 0) = 0$
$f_{2.21}$ - **Schwefel's Problem 2.21** $f_{2.21}(x) = \max\_i\{\lvert x_i \rvert, 1 \leq i \leq D\}$ $-100 \leq x_i \leq 100$ $\min(f(x^*)) = f_{2.21}(0, \dots, 0) = 0$	f_{stp} – **Step function** $f_{stp}(x) = \sum_{i=1}^{D} (\lfloor x_i + 0.5 \rfloor)^2$ $-100 \leq x_i \leq 100$ $\min(f(x^*)) = f_{stp}(0, \dots, 0) = 0$
f_{qtc} –**Quartic Function** $f_{qtc}(x) = \sum_{i=1}^{D} i x_i^4$ $-1.28 \leq x_i \leq 1.28$ $\min(f(x^*)) = f_{qtc}(0, \dots, 0) = 0$	$f_{1.2}$ – **Schwefel's Problem 1.2** $f_{1.2}(x) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} x_j \right)^2$ $-100 \leq x_i \leq 100$ $\min(f(x^*)) = f_{1.2}(0, \dots, 0) = 0$
f_{sch} - **Schwefel's Problem 2.26** $f_{sch}(x) = \sum_{i=1}^{D} \left(x_i sin(\sqrt{\lvert x_i \rvert}) \right)$ $-500 \leq x_i \leq 500$ $\min(f(x^*)) = f_{sch}(420.9687, \dots, 420.9687) = -420.9687D$	f_{ras} – **Rastrigin's function** $f_{ras}(x) = \sum_{i=1}^{D} (x_i^2 - 10\cos(2\pi x_i) + 10)$ $-5.12 \leq x_i \leq 5.12$ $\min(f(x^*)) = f_{ras}(0, \dots, 0) = 0$
f_{ros} –**Rosembrock's function** $f_{ros}(x) = \sum_{i=1}^{D-1} \lvert 100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \rvert$ $-30 \leq x_i \leq 30$ $\min(f(x^*)) = f_{ros}(1, \dots, 1) = 0$	f_{ack} – **Ackley's function** $f_{ack}(x) = 20 - 20exp\left(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - exp\left(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\right) + e$ $-32 \leq x_i \leq 32$ $\min(f(x^*)) = f_{ack}(0, \dots, 0) = 0$
f_{grw} –**Griewank's function** $f_{grw}(x) = \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$ $-600 \leq x_i \leq 600$ $\min(f(x^*)) = f_{grw}(0, \dots, 0) = 0$	f_{sal} – **Salomon's function** $f_{sal}(x) = 1 - \cos\left(2\pi\sqrt{f_{sph}(x)}\right) + 0.1\sqrt{f_{sph}(x)}$ $-100 \leq x_i \leq 100$ $\min(f(x^*)) = f_{sal}(0, \dots, 0) = 0$

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f_{pen1}, f_{pen2} **–Generalized penalized functions**

$$f_{pen1}(x) = \frac{\pi}{D}\left(10\sin^2(\pi y_1) - \sum_{i=1}^{D-1}((y_i - 1)^2\,(1 + 10\sin^2(\pi y_{i+1}))) + (y_D - 1)^2\right) + \sum_{i=1}^{D} u(x_i, 10{,}100{,}4)$$

$$-50 \leq x_i \leq 50$$

$$\min\big(f(x^*)\big) = f_{pen1}(1, \dots, 1) = 0$$

$$f_{pen2}(x) = 0.1\left(\sin^2(3\pi x_1) - \sum_{i=1}^{D-1}((x_i - 1)^2\,(1 + \sin^2(3\pi x_{i+1}))) + (X_D - 1)^2(1 + \sin^2(2\pi x_D))\right) + \sum_{i=1}^{D} u(x_i, 5{,}100{,}4)$$

$$-50 \leq x_i \leq 50$$

$$\min\big(f(x^*)\big) = f_{pen2}(1, \dots, 1) = 0$$

$$u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m, x_i > a. \\ 0, -a \leq x_i \leq a \\ k(-x_i - a)^m, x_i < -a. \end{cases}$$

$$y_i = 1 + \frac{1}{4}(x_i + 1)$$
